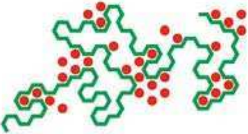
STATISTICS OF LINEAR POLYMERS IN DISORDERED MEDIA



Bikas K. Chakrabarti

EDITOR

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Edited by
Bikas K. Chakrabarti
Saha Institute of Nuclear Physics
Kolkata, India

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PREFACE

With the mapping of the partition function graphs of the n-vector magnetic model in the $n \to 0$ limit as the self-avoiding walks, the conformational statistics of linear polymers was clearly understood in early 1970s. Various models of disordered solids, percolation model in particular, were also established by late seventies. Subsequently, investigations on the statistics of linear polymers or of self-avoiding walks in, say, porous medium or disordered lattices were started in early 1980s. Inspite of the brilliant ideas forwarded and extensive studies made, the problem is not yet completely solved in its generality. This intriguing and important problem has remained since a topic of vigorous and active research.

This book intends to offer the readers a first hand and extensive review of the various aspects of the problem, written by the experts in the respective fields. S. M. Bhattacharjee has reviewed the success in dealing with the directed polymers in random medium and has also discussed the problem of unzipping of a pair of directed chains where disorder appears along the chains. A. J. Guttmann has reviewed the series studies of self-avoiding walk statistics for various constrained and random geometries, including the problem of unzipping of the DNA chains. V. Blavats'ka, C. von Ferber, R. Folk and Yu. Holovatch has reviewed extensively the field theoretic and real space renormalization group studies for the problem, including the effects of correlated disorder. D. Dhar and Y. Singh have reviewed most of the exact results for self-avoiding walks on different non-random fractals, including various simplex lattices, employing the real space renormalization group technique. A. Ordemann, M. Porto and H. E. Roman have reviewed the extensive numerical studies on self-avoiding walks on various deterministic and random fractals; percolation clusters in particular. In absence of strict self-avoiding restriction, the analogy of the problem with that of quantum particles in disorder, and the consequent localization of the polymers in random media, have been reviewed by Y. Y. Goldschmidt and Y. Shiferaw. P. Bhattacharyya and A. Chatterjee have reviewed the properties of various optimal and most probable (self-avoiding in general) paths on randomly disordered lattices, including the statistics of the Travelling Salesman Problem on dilute lattices. Finally, G. D. J. Phillies has given an extensive overview of the experimental studies on polymer diffusion in random environments of the solutions.

We earnestly hope, the contents of the book will provide a valuable guide for researchers in statistical physics of polymers and will surely induce further research and advances towards a complete understanding of the problem.

I am grateful to all the contributors for their wonderful contributions and cooperations. I am thankful to Arnab Chatterjee for his help in the compilation of the book.

Bikas K. Chakrabarti
Theoretical Condensed Matter Physics Division, and
Centre for Applied Mathematics & Computational Science
Saha Institute of Nuclear Physics, Kolkata 700064, India.

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Contents

Statistics of Linear Polymers in Disordered Media
Edited by Bikas K. Chakrabarti

Linear and branched polymers on fractals

Deepak Dhar[a] and Yashwant Singh [b]

[a] Department of Theoretical Physics,
Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400005, India

[b]Department of Physics,
Banaras Hindu University,
Varanasi, U.P. 221005, India.

This is a pedagogical review of the subject of linear polymers on deterministic finitely ramified fractals. For these, one can determine the critical properties exactly by real-space renormalization group technique. We show how this is used to determine the critical exponents of self-avoiding walks on different fractals. The behavior of critical exponents for the n-simplex lattice in the limit of large n is determined. We study self-avoiding walks when the fractal dimension of the underlying lattice is just below 2. We then consider the case of linear polymers with attractive interactions, which on some fractals leads to a collapse transition. The fractals also provide a setting where the adsorption of a linear chain near on attractive substrate surface and the zipping-unzipping transition of two mutually interacting chains can be studied analytically. We also discuss briefly the critical properties of branched polymers on fractals.

1. INTRODUCTION

The problem of self-avoiding walks (SAWs) on lattices provides perhaps the simplest geometrical model of equilibrium critical phenomena (i.e., non-trivial power-laws in the behavior of different quantities in a system in thermal equilibrium). The two other familiar examples of geometrical models showing phase transitions, the percolation problem and a system of hard particles (spheres or rods), both involve more complex geometrical structures. The model of SAWs captures the important macroscopic features of polymers in solution, and is closely related to other models of phase-transitions in statistical physics like the Ising model [1], and can also be seen as the $n \to 0$ limit of the n-vector model [2]. Given the many technological applications of polymers, and the importance of SAWs as a model of critical phenomena, it is not surprizing that the model has attracted a lot of attention in the last sixty years. Several good reviews are available summarizing our current understanding of this problem [3].

The SAW problem is clearly trivial in one dimension. In spite of the large number of papers related to this problem, an exact solution of the problem has not been possible so far, for any non-trivial case. In two dimensions, the exact value of the growth constant is

Solving the SAW problem with quenched disorder is another interesting question. For the 3-simplex fractal, this corresponds to making the variables $B^{(r)}$ random variables, and one has to determine the probability distribution of this variable for large r.

We have discussed only the equilibrium properties of polymers. Of course, in many real systems, the time scales for equilibriation can be very large. It is thus of interest to study non-equilibrium properties of statistical mechanical systems on fractals. A simple prototype is the study of kinetic Ising model on fractals. Closer to our interests here, one can study, say, the reptation motion of a polymer on the fractal substrate. This seems to be a rather good first model of motion of a polymer in gels.

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