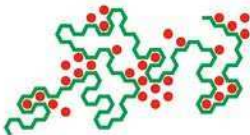




STATISTICS OF LINEAR POLYMERS IN DISORDERED MEDIA



Bikas K. Chakrabarti
EDITOR

Statistics of Linear Polymers in Disordered Media

Edited by

Bikas K. Chakrabarti

Saha Institute of Nuclear Physics
Kolkata, India

2005



ELSEVIER

Amsterdam – Boston – Heidelberg – London – New York – Oxford
Paris – San Diego – San Francisco – Singapore – Sydney – Tokyo

ELSEVIER B.V.
Radarweg 29
P.O. Box 211, 1000 AE Amsterdam
The Netherlands

ELSEVIER Inc.
525 B Street, Suite 1900
San Diego, CA 92101-4495
USA

ELSEVIER Ltd
The Boulevard, Langford Lane
Kidlington, Oxford OX5 1GB
UK

ELSEVIER Ltd
84 Theobalds Road
London WC1X 8RR
UK

© 2005 Elsevier B.V. All rights reserved.

This work is protected under copyright by Elsevier B.V., and the following terms and conditions apply to its use:

Photocopying

Single photocopies of single chapters may be made for personal use as allowed by national copyright laws. Permission of the Publisher and payment of a fee is required for all other photocopying, including multiple or systematic copying, copying for advertising or promotional purposes, resale, and all forms of document delivery. Special rates are available for educational institutions that wish to make photocopies for non-profit educational classroom use.

Permissions may be sought directly from Elsevier's Rights Department in Oxford, UK: phone (+44) 1865 843830, fax (+44) 1865 853333, e-mail: permissions@elsevier.com. Requests may also be completed on-line via the Elsevier homepage (<http://www.elsevier.com/locate/permissions>).

In the USA, users may clear permissions and make payments through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA; phone: (+1) (978) 7508400, fax: (+1) (978) 7504744, and in the UK through the Copyright Licensing Agency Rapid Clearance Service (CLARCS), 90 Tottenham Court Road, London W1P 0LP, UK; phone: (+44) 20 7631 5555; fax: (+44) 20 7631 5500. Other countries may have a local reprographic rights agency for payments.

Derivative Works

Tables of contents may be reproduced for internal circulation, but permission of the Publisher is required for external resale or distribution of such material. Permission of the Publisher is required for all other derivative works, including compilations and translations.

Electronic Storage or Usage

Permission of the Publisher is required to store or use electronically any material contained in this work, including any chapter or part of a chapter.

Except as outlined above, no part of this work may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without prior written permission of the Publisher. Address permissions requests to: Elsevier's Rights Department, at the fax and e-mail addresses noted above.

Notice

No responsibility is assumed by the Publisher for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions or ideas contained in the material herein. Because of rapid advances in the medical sciences, in particular, independent verification of diagnoses and drug dosages should be made.

First edition 2005

Library of Congress Cataloging in Publication Data

A catalog record is available from the Library of Congress.

British Library Cataloguing in Publication Data

A catalogue record is available from the British Library.

ISBN: 0-444-51709-X

∞ The paper used in this publication meets the requirements of ANSI/NISO Z39.48-1992 (Permanence of Paper).
Printed in The Netherlands.

PREFACE

With the mapping of the partition function graphs of the n -vector magnetic model in the $n \rightarrow 0$ limit as the self-avoiding walks, the conformational statistics of linear polymers was clearly understood in early 1970s. Various models of disordered solids, percolation model in particular, were also established by late seventies. Subsequently, investigations on the statistics of linear polymers or of self-avoiding walks in, say, porous medium or disordered lattices were started in early 1980s. In spite of the brilliant ideas forwarded and extensive studies made, the problem is not yet completely solved in its generality. This intriguing and important problem has remained since a topic of vigorous and active research.

This book intends to offer the readers a first hand and extensive review of the various aspects of the problem, written by the experts in the respective fields. S. M. Bhattacharjee has reviewed the success in dealing with the directed polymers in random medium and has also discussed the problem of unzipping of a pair of directed chains where disorder appears along the chains. A. J. Guttmann has reviewed the series studies of self-avoiding walk statistics for various constrained and random geometries, including the problem of unzipping of the DNA chains. V. Blavats'ka, C. von Ferber, R. Folk and Yu. Holovatch has reviewed extensively the field theoretic and real space renormalization group studies for the problem, including the effects of correlated disorder. D. Dhar and Y. Singh have reviewed most of the exact results for self-avoiding walks on different non-random fractals, including various simplex lattices, employing the real space renormalization group technique. A. Ordemann, M. Porto and H. E. Roman have reviewed the extensive numerical studies on self-avoiding walks on various deterministic and random fractals; percolation clusters in particular. In absence of strict self-avoiding restriction, the analogy of the problem with that of quantum particles in disorder, and the consequent localization of the polymers in random media, have been reviewed by Y. Y. Goldschmidt and Y. Shiferaw. P. Bhattacharyya and A. Chatterjee have reviewed the properties of various optimal and most probable (self-avoiding in general) paths on randomly disordered lattices, including the statistics of the Travelling Salesman Problem on dilute lattices. Finally, G. D. J. Phillies has given an extensive overview of the experimental studies on polymer diffusion in random environments of the solutions.

We earnestly hope, the contents of the book will provide a valuable guide for researchers in statistical physics of polymers and will surely induce further research and advances towards a complete understanding of the problem.

I am grateful to all the contributors for their wonderful contributions and cooperations. I am thankful to Arnab Chatterjee for his help in the compilation of the book.

Bikas K. Chakrabarti
Theoretical Condensed Matter Physics Division, and
Centre for Applied Mathematics & Computational Science
Saha Institute of Nuclear Physics, Kolkata 700064, India.

January 2005.

Contents

Polymers in random media: An introduction <i>B. K. Chakrabarti</i>	1
Directed polymers and randomness <i>S. M. Bhattacharjee</i>	9
Self-avoiding walks in constrained and random geometries: Series studies <i>A. J. Guttmann</i>	59
Renormalization group approaches to polymers in disordered media <i>V. Blavats'ka, C. von Ferber, R. Folk and Yu. Holovatch</i>	103
Linear and branched polymers on fractals <i>D. Dhar and Y. Singh</i>	149
Self-avoiding walks on deterministic and random fractals: Numerical results <i>A. Ordemann, M. Porto and H. E. Roman</i>	195
Localization of polymers in random media: Analogy with quantum particles in disorder <i>Y. Y. Goldschmidt and Y. Shiferaw</i>	235
Geometric properties of optimal and most probable paths on randomly disordered lattices <i>P. Bhattacharyya and A. Chatterjee</i>	271
Phenomenology of polymer single-chain diffusion in solution <i>G. D. J. Phillies</i>	305
Index	357

Linear and branched polymers on fractals

Deepak Dhar^a and Yashwant Singh^b

^a Department of Theoretical Physics,
Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400005, India

^bDepartment of Physics,
Banaras Hindu University,
Varanasi, U.P. 221005, India.

This is a pedagogical review of the subject of linear polymers on deterministic finitely ramified fractals. For these, one can determine the critical properties exactly by real-space renormalization group technique. We show how this is used to determine the critical exponents of self-avoiding walks on different fractals. The behavior of critical exponents for the n -simplex lattice in the limit of large n is determined. We study self-avoiding walks when the fractal dimension of the underlying lattice is just below 2. We then consider the case of linear polymers with attractive interactions, which on some fractals leads to a collapse transition. The fractals also provide a setting where the adsorption of a linear chain near on attractive substrate surface and the zipping-unzipping transition of two mutually interacting chains can be studied analytically. We also discuss briefly the critical properties of branched polymers on fractals.

1. INTRODUCTION

The problem of self-avoiding walks (SAWs) on lattices provides perhaps the simplest geometrical model of equilibrium critical phenomena (i.e., non-trivial power-laws in the behavior of different quantities in a system in thermal equilibrium). The two other familiar examples of geometrical models showing phase transitions, the percolation problem and a system of hard particles (spheres or rods), both involve more complex geometrical structures. The model of SAWs captures the important macroscopic features of polymers in solution, and is closely related to other models of phase-transitions in statistical physics like the Ising model [1], and can also be seen as the $n \rightarrow 0$ limit of the n -vector model [2]. Given the many technological applications of polymers, and the importance of SAWs as a model of critical phenomena, it is not surprising that the model has attracted a lot of attention in the last sixty years. Several good reviews are available summarizing our current understanding of this problem [3].

The SAW problem is clearly trivial in one dimension. In spite of the large number of papers related to this problem, an exact solution of the problem has not been possible so far, for any non-trivial case. In two dimensions, the exact value of the growth constant is

Solving the SAW problem with quenched disorder is another interesting question. For the 3-simplex fractal, this corresponds to making the variables $B^{(r)}$ random variables, and one has to determine the probability distribution of this variable for large r .

We have discussed only the equilibrium properties of polymers. Of course, in many real systems, the time scales for equilibration can be very large. It is thus of interest to study non-equilibrium properties of statistical mechanical systems on fractals. A simple prototype is the study of kinetic Ising model on fractals. Closer to our interests here, one can study, say, the reptation motion of a polymer on the fractal substrate. This seems to be a rather good first model of motion of a polymer in gels.

Acknowledgements: It is a pleasure to thank Sumedha for a careful reading of the manuscript, and her many constructive suggestions for an improved presentation.

REFERENCES

1. M.E. Fisher and M. F. Sykes, Phys. Rev. **114** (1959) 45.
2. P. G. de Gennes, Scaling Concepts in Polymer Physics, Cornell University Press, Ithaca, 1979.
3. N. Madras and G. Slade, The Self-Avoiding Walk, Birkhauser, Boston, 1993; D. S. McKenzie, Phys. Rep. C **27** (1976) 35; Polymer Physics: 25 Years of the Edwards Model, Ed. S. M. Bhattacharjee, World Scientific, Singapore, 1992; K. Y. Lin and Y. C. Hsiao, Chinese J. Phys. **31** (1993) 695; A. J. Guttmann, in Computer Aided Statistical Physics, Ed. C. K. Hu, A.I.P. Conf. Proc. Vol. 248, (1992) 34.
4. B. Nienhuis, Phys. Rev. Lett. **49** (1982) 1062.
5. I. Jensen and A. J. Guttmann, J. Phys. A: Math. Gen. **32** (1999) 4867.
6. B. Duplantier and H. Saleur, Nucl. Phys. B **290** (FS 20) (1987) 291.
7. T. Hara and G. Slade, Comm. Math. Phys. **147** (1992) 101; *ibid* Rev. Math. Phys. **4**(1992) 235.
8. K. Wilson and J. Kogut, Phys. Rep. C **12** (1974) 75.
9. F. E. Stillinger, J. Math. Phys. **18** (1977) 1244.
10. D. Dhar, in Polymer Physics: 25 years of the Edwards' Model, Ed. S. M. Bhattacharjee, World Scientific, Singapore, 1992, p. 83.
11. D. R. Nelson and M. E. Fisher, Ann. Phys.(NY) **91** (1975) 226.
12. D. Dhar, J. Math. Phys. **18** (1977) 577.
13. J. A. Given and B. B. Mandelbrot, J. Phys. A **16** (1983) L565.
14. D. Dhar, J. Phys. A **21** (1988) 2261.
15. R. Hilfer and A. Blumen, J. Phys. A: Math. Gen. **17** (1984) L537.
16. S. Milosevic, D. Stassinopoulos and H. E. Stanley, J. Phys. A **21** (1988) 1477.
17. D. Dhar, J. Math. Phys. **19** (1978) 5.
18. R. Rammal G. Toulouse and J. Vannimenus, J. Phys. (Paris) **45** (1984) 389.
19. T. Hattori and T. Tsuda, J. Stat. Phys. **109** (2002) 39.
20. D. Sornette, Phys. Rep. **297** (1998) 239.
21. S. Milosevic, I Zivic, and S. Elezovic Hadzic, Phys. Rev. E **61** (2000) 2141.
22. Sumedha and D. Dhar, (2004), in preparation.
23. S. Kumar, Y. Singh and Y. P. Joshi, J. Phys. A: Math. Gen. **23** (1990) 2987.
24. S. Kumar and Y. Singh, J. Phys. A: Math. Gen. **23** (1990) 5115

25. J. R. Melrose, *J. Phys. A : Math. and Gen.* **18** (1985), L17; and references cited therein.
26. S. Elezovic, M. Knezevic and S. Milosevic, *J. Phys. A* **20** (1987) 1215.
27. V. Bujanja and M. Knezevic, cited in [30].
28. D. Dhar, *J. Phys. (Paris)*, **49** (1988) 397.
29. J. Cardy and S. Redner, *J. Phys A: Math and Gen. A* **17** (1984) L933.
30. S. Milosevic and I Zivic, *J. Phys A* **24** (1991) L833.
31. I. Zivic and S. Milosevic, *J. Phys. A* **26** (1993) 3393.
32. A. J. Guttmann and G. M. Torrie, *J. Phys. A: Math and Gen* **17** (1984) 3541.
33. K. Tanaka, *Sci. Am.* **244**(1) (1981) 110; J. des Cloiseaux and G. Jannink; *Polymers in Solutions*, Clarendon Press, Oxford, 1990.
34. D. J. Klein and W. A. Seitz, *J. Physique Lett.* **45** (1984) L241.
35. D. Dhar and J. Vannimenus, *J. Phys. A: Math. Gen.* **20**, (1987) 199.
36. S. Kumar and Y. Singh, *Phys. Rev. A* **42** (1990) 7151.
37. M. Knezevic and J. Vannimenus, *J. Phys. A: Math.Gen.* **20** (1987) L969.
38. A. Malakis, *J. Phys. A: Math. Gen.* **9** (1976) 1283.
39. E. Orlandini, F. Seno, A. L. Stella and M. C. Tesi *Phys. Rev. Lett.* **68** (1992) 488.
40. A. B. Harris and T. C. Lubensky, *Phys. Rev. B* **24**(1981) 2656; T. C. Lubensky and J. Isaacson, *Phys. Rev. Lett.* **41**(1978) 829, *Phys. Rev.A* **20** (1979) 2130.
41. G. Parisi and N. Sourlas, *Phys. Rev. Lett.* **46** (1981) 871; D. C. Bridges and J. Z. Imbrie, *Ann. Math.* **158** (2003) 1019.
42. M. Knezevic and J. Vannimenus, *Phys. Rev. Lett.* **56** (1986) 1591.
43. D. Dhar and A. Dhar, *Phys. Rev. E* **55** (1997) R 2093.
44. M. Knezevic and J. Vannimenus, *Phys. Rev. B* **35** (1987) 4988.
45. B. Derrida and H. Herrmann, *J. Phys.(Paris)* **44** (1983) 1365.
46. D. Dhar, (2004), submitted for publication.
47. K. De'Bell and T. Lookman, *Rev. Mod. Phys.* **65** (1993) 87.
48. R. Rajesh, D. Dhar, D. Giri, S. Kumar and Y. Singh, *Phys. Rev. E* **65** (2002) 056124.
49. Y. Singh, S. Kumar and D. Giri, *J. Phys. A: Math. Gen* **32**, L407, (1999); **34** (2001) L67.
50. P. Grassberger and R. Hegger, *Phys. Rev. E* **51** (1995) 2674.
51. A. R. Veal, J.M. Yeomans and G. J. Jug, *J. Phys. A: Math. Gen* **24** (1991) 827.
52. S. Kumar and Y. Singh, *Physica A* **229** (1996) 61.
53. T. W. Burkhardt, E. Eisenriegler and I. Guim, *Nucl. Phys. B* **316** (1989) 559.
54. E. Bouchaud and J. Vannimenus, *J. Physique* **50** (1989) 2931.
55. S. Kumar, Y. Singh and D. Dhar, *J. Phys. A:Math. Gen.* **26** (1993) 4835.
56. V. Bujanja, M. Knezevic and J. Vannimenus, *J. Stat. Phys.* **71** (1993) 1.
57. I. Zivic, S. Milosevic and H.E. Stanley, *Phys. Rev. E* **49** (1994) 636.
58. T. Ischinabe, *J. Chem. Phys.* **80** (1984) 1318.
59. Y. Singh, S. Kumar and D. Giri, *Pramana J. of Phys.* **53** (1999) 37.
60. S. Kumar and Y. Singh, *Phys. Rev. E* **48** (1993) 734.
61. S. Kumar and Y. Singh, *J. Phys. A:Math and Gen.* **26** (1993) L987.
62. S. Kumar and Y. Singh, *Phys. Rev. E* **51** (1995) 579.
63. S. Kumar and Y. Singh, *J. Stat. Phys.* **89** (1997) 981.
64. A. Ordemann, M. Porto, and H. E. Roman, *Phys. Rev. E* **65** (2002) 021107.